

Formalizing Addition in Quadrature with Computational Applications

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1 Overview

My overall plan is to:

- introduce the basic precepts of what Adding in Quadrature is
- explain how it applies to spreadsheet models
- identify some key theorems to be proven
- outline the basic proof strategy
- demonstrate some computations premised on the theorems using data taken directly from Chapter 8 of my book (*Excel 2003 Best Practices for Business* - Wiley Publishing, Inc. ISBN: 076454120X)

2 Core Property

Theorems will prove that quadrature gives smaller results than linear addition; thereby providing a conservative (safe) upper bound for combined uncertainty.

3 Challenges

Challenge 1:

The mathematical expressions involved with Adding in Quadrature is not terribly difficult. Arriving at a proof for much of it is straightforward enough.

Separately, incorporating it within numerical calculations is very easy to apply. But there is a “slight of hand” that takes place. The core theorems are all built with Reals, so definitions involving Reals are inherently noncomputable (they can’t be directly used in `#eval` expressions). But their sister definitions used in numerical computations appear almost identical in structure, except they involve Floats.

My challenge is to come up with a framework so that it is unnecessary to independently write two sets of equations or definitions that are essentially identical (one for Reals and the

other for Floats). The only thing linking them is faith that they match. They need to be wedded in a direct way.

Challenge 2:

The second challenge is that I want to unequivocally prove that the numerical computations involving Floats directly and rigorously connect back to the core theorems that underpin Adding in Quadrature. That is, the core theorems form an integral part of the numerical applications of mathematical modeling.

From a computational performance standpoint Lean may not be particularly performant. What it would accomplish is to place the validity of applying the framework of Adding in Quadrature onto a sound and solid bedrock.

In my mind, achieving this would greatly extend the reach of formalizing areas of mathematics from (elegant) theorem proving well into the realm of concrete applications.